# Fast and Efficient Detection of Edges: Dynamic Programming and Subsampling

Mathematics Applied to Image Processing

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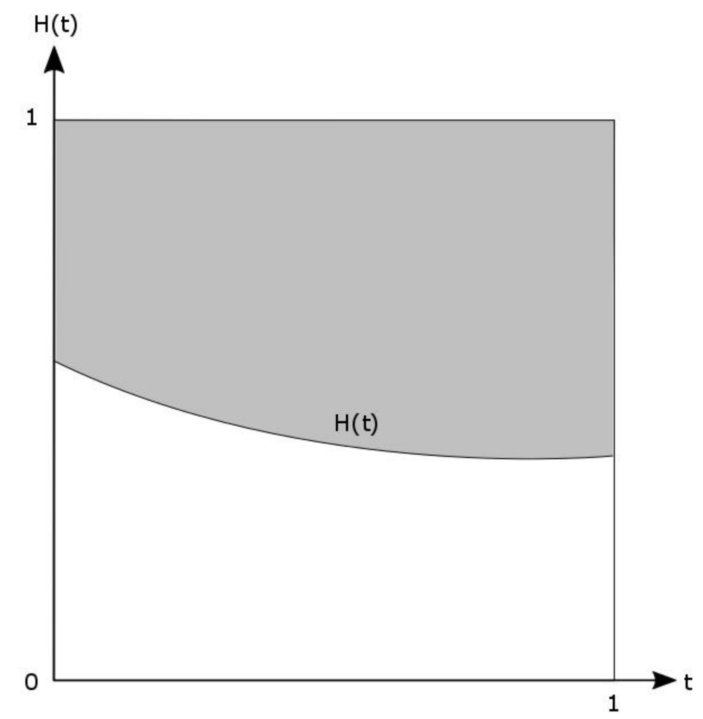
## 1. Problem

How to detect and localize horizon functions H(t) fastly and efficiently in an image?




## 2. Assumptions

- We consider the noise as an additive gaussian random value.
- We assume that in the noiseless image, the function is piecewise constant, with different constants above and below the horizon curve.
- The horizon function is regular enough – it respects the Hölder condition – and is far enough from the borders on the top and the bottom of the image (as defined by Korostelev and Tsybakov in [1]).

## 3. Presentation of the project

Many algorithms have been developped over the last decades. We here introduce a new algorithm that is both fast – sublinear – and efficient.

Our algorithm is based on a multiscale edgelets acyclic directed graph structure. It runs an optimization routine that uses the concept of dynamic programming to find the best path of edgelets. We basically use the concept introduced in [2] for the detection of chirps.

Another version of this algorithm follows the same process but only runs the subsampled image by considering only certain entire columns, as proposed in [3]. This algorithm is faster than the previous one, while appearing to be a little less accurate.

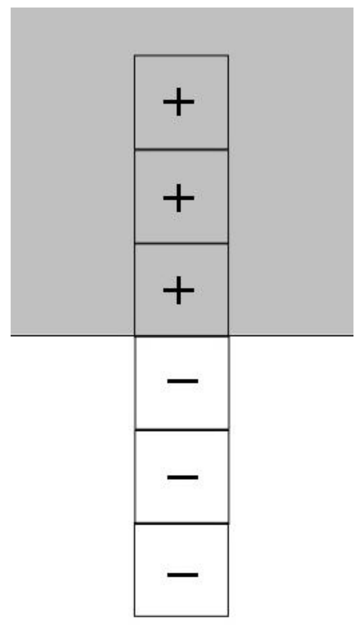
## 4. Structure of the algorithm

| Subsampling | Edgelet transform | Best path algorithm | Detection |
|---|---|---|---|
| We only consider certain entire columns of the image. | Computation of the edgelet transform – i.e. response of all the edgelets of the dictionary. | Best path algorithm – routine that finds out the best acyclic path of edgelets by using dynamic programming. | Detection – the best path is considered only if it is statistically significant. |

## 5. Edgelet dictionary & edgelet transform

The edgelet dictionary is the set of the $O(\log(n)n^2)$ multiscale edgelets that we consider and that can potentially approximate the horizon function. We only consider edgelets whose slope is equal or lower to 1 (in absolute value).

We compute the edgelet transform by computing the responses $R$ of all the edgelets of the dictionary. An edgelet response is the weighted sum of the responses of the pixels along the edgelet to the mask.




Mask

## 6. Graph structure

We consider the directed acyclic graph $G(W, E)$ made up by the set of nodes $W = v_1, \ldots, v_{|V|}$ and the set of edges $E$ as detailed below :

| In the image | In the graph structure |
|---|---|
| edgelet | node |
| Potential connection between two edgelets | edge |

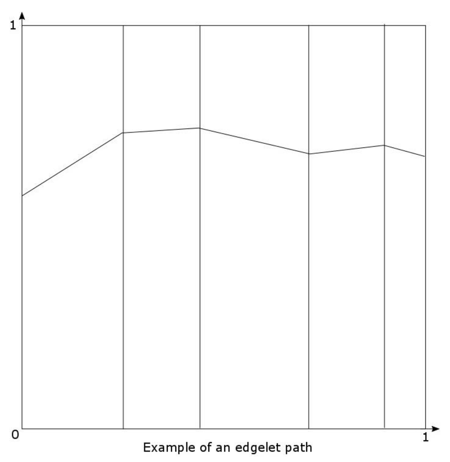
We define the cost function $C : E \rightarrow \mathbb{R}_+$ on the edges in the graph as following : $C(v_i\ ,\ v_j) = R(v_j\ )$, the response of the edgelet corresponding to the node $v_j$.

## 7. Best path algorithm

This is the optimization routine – based on dynamic programming - that finds the best path of edgelets $W^*$ in the graph algorithm by solving:

$$W^* = \max_{W} \left| \sum_{v_i, v_j \in W \text{ and } (v_i,\, v_j) \in E} C(v_i,\, v_j) \right|$$

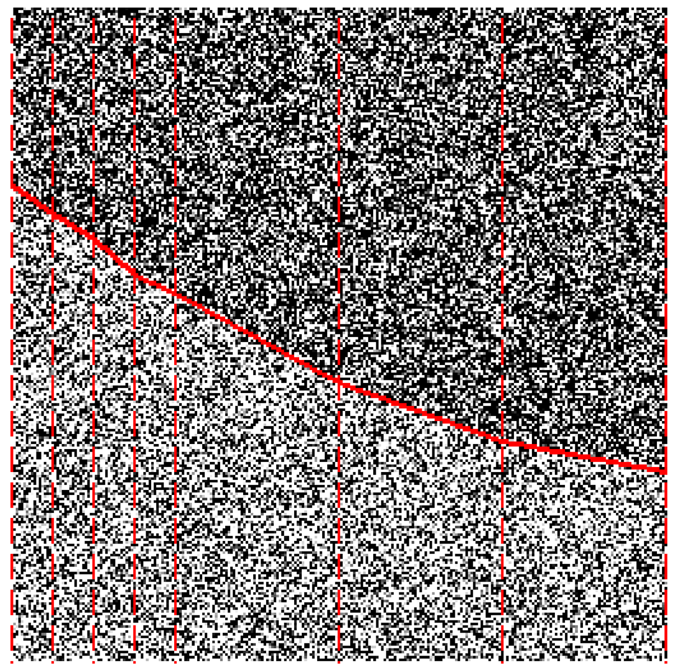
Only edges corresponding to a good continuation between two nodes are considered.



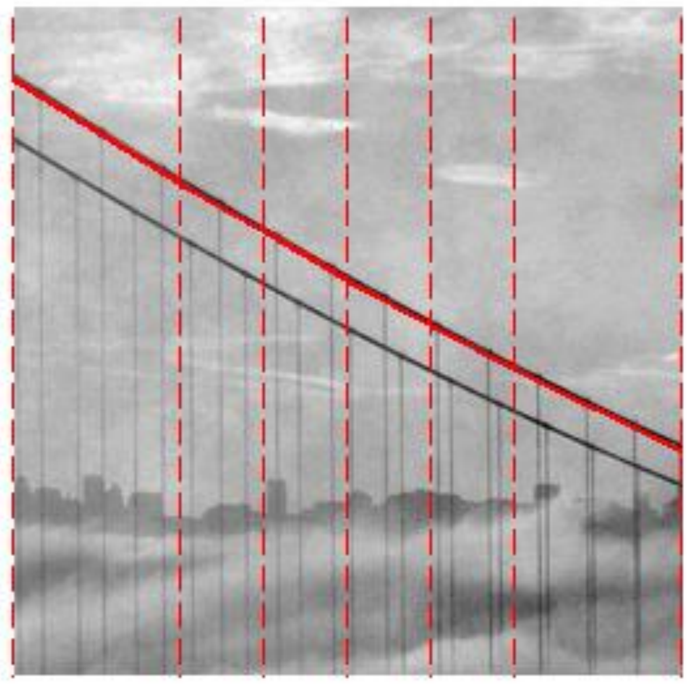

## 8. Experiments

Noiseless image

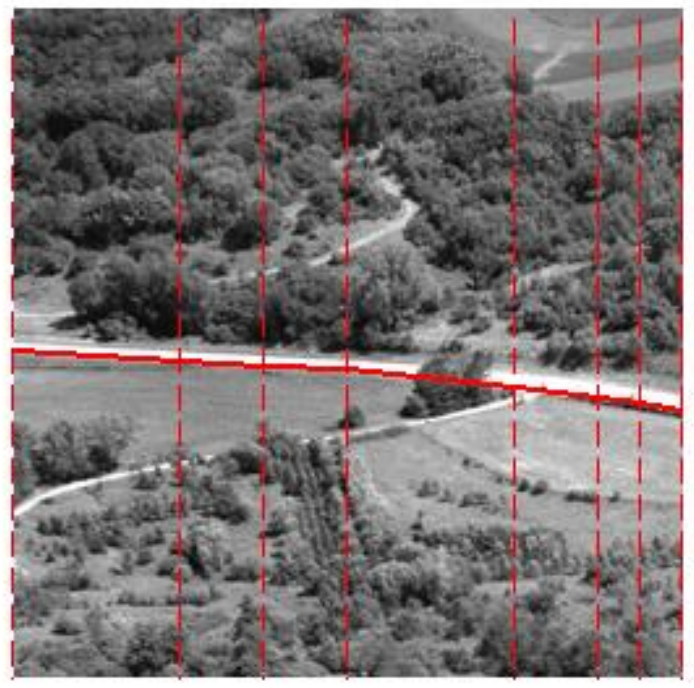
Best path restricted to #edgelets=7, |cost|=1.0534



Best path restricted to #edgelets=6, |cost|=15.3843



Best path restricted to #edgelets=7, |cost|=18.2311



## 9. Conclusion

- Both fast and efficient algorithm.
- The threshold is only applied to the best path, and not to each edgelet. It is then able to detect partly hidden horizon curves.
- Tradeoff between approximation quality and variance error : lot of short edgelets will approximate more accuratly the horizon curve than a few long ones, but the error due to the variance will be higher.

## 10. Bibliography


- [1] Korostelev and Tsybakov. *Minimax Theory of Image Reconstruction*, volume 82. Springer, 1993.
- [2] Emmanuel J Candes et al. Detecting highly oscillatory signals by chirplet path pursuit. *Applied and Computational Harmonic Analysis*, 24(1):14-40, 2008
- [3] Ery Arias-Castro et al. Edge detection on a computational budget: a sub-linear approach, 2014.