Consistent estimation of redundant signal representations

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Abstract :

After transmission in an AWGN channel, MAP estimation can be inconsistent.
The goal of the internship was to develop a consistency test with the parity matrix, **P**, and to apply it to oversampled filter banks.
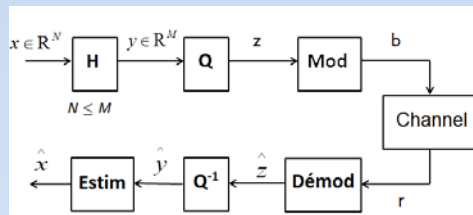
Fig.1 Joint source-channel scheme

MAP estimator :

$$\hat{\mathbf{x}}_{MAP} = \arg \max_{\mathbf{x} \in \mathbb{R}^N} p_{\mathbf{X}}(\mathbf{x}) \prod_{m=1}^{M} p_{R|Z}\left(r_m | q\left(\mathbf{h}_m^T \mathbf{x}\right)\right)$$

$$\exists \mathbf{x} \text{ tq } \mathbf{z} = Q(\mathbf{H}\mathbf{x})$$
$$\Longleftrightarrow$$
$$\exists \mathbf{y} \text{ tq } \mathbf{P}\mathbf{y} = \mathbf{0} \text{ et } \mathbf{z} = Q(\mathbf{y})$$

Consistency property

We try to estimate the input signal from the measure of quantized output, which passed through a noisy channel. A consistency test *was proposed* in a thesis [1] using interval analysis, but the pessimism reduce the performances.

Work :

The first step was to find a better characterization of the image of a hypercube under **P** :
- Described as the intersection of subspaces bounded by hyperplanes
- Using the model by zonotopes => iterative algorithm GJK [2]

We chose the zonotope characterization because the complexity was lower.

The second step was to improve the method to select the candidates for the consistency test :
- Try to estimate the number of wrong bits and the wrong bits among the total number of bits transmitted

Several simulations have been done to compare the performances of the algorithm with and without the consistency test and between the different proposed tests.
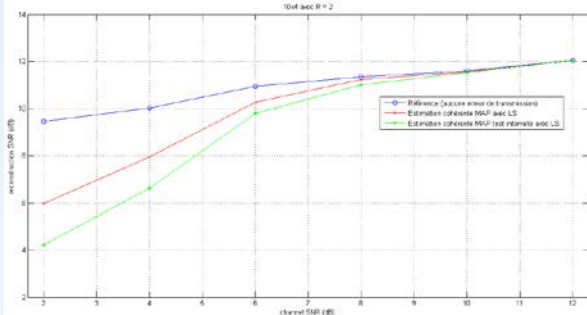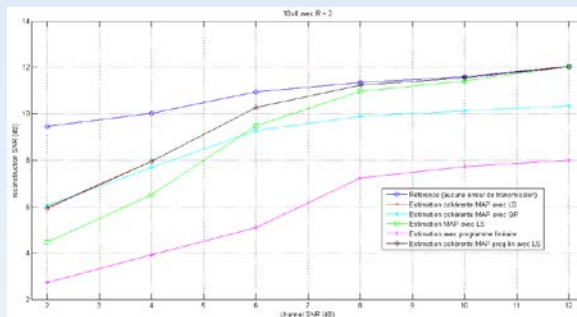

Fig.2 Comparison of different methods

Fig.3 Comparison of different tests

The final step was to apply the algorithm to oversampled filter banks [3] :
- Implement iteratively the algorithm
- Keep different decoding assumptions in the algorithm

Results and conclusion :

- Simulations with **H** a 10x4 matrix, ρ = 2 bits and 10000 noise realizations.
- Consistent MAP estimation has a better reconstruction SNR for small channel SNR than classical MAP estimation. (Fig.2)
- Zonotope test is better than interval test. (Fig.3) but slower.
- Not enough time to fully debug for the oversampled filter banks application.

References :

[1] Manel Abid. *Joint source-channel coding/decoding of multimedia contents*, Télécom ParisTech, 2012.
[2] G van den Bergen. *A fast and robust GJK implementation for collision detection of convex objects, Journal of graphics tools*, 1999.
[3] F. Labeau et al. Oversampled filter banks as error correcting codes : Theory and impulse noise correction, *IEEE*, 2005.